# Mathematica 11.3 Integration Test Results

## Test results for the 4 problems in "1.2.2.8 P(x) (d+e x)^q (a+b x^2+c x^4)^p.m"

### Problem 1: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e\,x)\,\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 405 leaves, 7 steps):

$$\frac{e\,\text{ArcTan}\left[\frac{\sqrt{-c\,d^4-a\,e^4}\;x}{d\,e\,\sqrt{a+c\,x^4}}\right]}{2\,\sqrt{-c\,d^4-a\,e^4}}-\frac{e\,\text{ArcTanh}\left[\frac{a\,e^2+c\,d^2\,x^2}{\sqrt{c\,d^4+a\,e^4}\;\sqrt{a+c\,x^4}}\right]}{2\,\sqrt{c\,d^4+a\,e^4}}+\frac{c^{1/4}\,d\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\left(\sqrt{c}\;d^2+\sqrt{a}\;e^2\right)\sqrt{a+c\,x^4}}-\frac{\left(\sqrt{c}\;d^2-\sqrt{a}\;e^2\right)\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{c}\;d^2+\sqrt{a}\;e^2\right)^2}{4\,\sqrt{a}\;\sqrt{c}\;d^2\,e^2},\,2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{1/4}\,c^{1/4}\,d\left(\sqrt{c}\;d^2+\sqrt{a}\;e^2\right)\sqrt{a+c\,x^4}}$$

Result (type 4, 200 leaves):

$$\left(\sqrt{1+\frac{c\,x^4}{a}}\left(-2\,(-1)^{1/4}\,a^{1/4}\sqrt{1+\frac{c\,d^4}{a\,e^4}}\;e\;\text{EllipticPi}\left[\frac{i\,\sqrt{a}\;e^2}{\sqrt{c}\;d^2},\,\text{ArcSin}\left[\frac{(-1)^{3/4}\,c^{1/4}\,x}{a^{1/4}}\right],-1\right]+c^{1/4}\,d\,\text{Log}\left[\frac{-d^2+e^2\,x^2}{c\,d^2\,x^2+a\,e^2\left(1+\sqrt{1+\frac{c\,d^4}{a\,e^4}}\;\sqrt{1+\frac{c\,x^4}{a}}\right)}\right]\right)\right)\Bigg/\left(2\,c^{1/4}\,d\,\sqrt{1+\frac{c\,d^4}{a\,e^4}}\;e\,\sqrt{a+c\,x^4}\right)$$

## Problem 2: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e\,x)^2\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 610 leaves, 11 steps):

$$-\frac{e^3\sqrt{a+c\,x^4}}{\left(c\,d^4+a\,e^4\right)(d+e\,x)}+\frac{\sqrt{c}\;e^2\,x\sqrt{a+c\,x^4}}{\left(c\,d^4+a\,e^4\right)\left(\sqrt{a}+\sqrt{c}\;x^2\right)}-\frac{c\,d^3\,e\,\text{ArcTan}\left[\frac{\sqrt{-c\,d^4-a\,e^4}\;x}{d\,e\sqrt{a+c\,x^4}}\right]}{\left(-c\,d^4-a\,e^4\right)^{3/2}}-\frac{c\,d^3\,e\,\text{ArcTanh}\left[\frac{a\,e^2+c\,d^2\,x^2}{\sqrt{c\,d^4+a\,e^4}\;\sqrt{a+c\,x^4}}\right]}{\left(c\,d^4+a\,e^4\right)^{3/2}}-$$

$$\frac{a^{1/4}\,c^{1/4}\,e^2\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{\left(c\,d^4+a\,e^4\right)\sqrt{a+c\,x^4}}+$$

$$\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\left(\sqrt{c}\;d^2+\sqrt{a}\;e^2\right)\sqrt{a+c\,x^4}}-$$

$$\left(c^{3/4}\,d^2\left(\sqrt{c}\;d^2-\sqrt{a}\;e^2\right)\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{c}\;d^2+\sqrt{a}\;e^2\right)^2}{4\sqrt{a}\;\sqrt{c}\;d^2\,e^2},\right.\right.$$

$$\left.\left.2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(2\,a^{1/4}\left(\sqrt{c}\;d^2+\sqrt{a}\;e^2\right)\left(c\,d^4+a\,e^4\right)\sqrt{a+c\,x^4}\right)$$

Result (type 4, 462 leaves):

$$\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(c\,d^4+a\,e^4\right)^{3/2}(d+e\,x)\sqrt{a+c\,x^4}}$$

$$\left(\sqrt{a}\,\sqrt{c}\,e^2\sqrt{c\,d^4+a\,e^4}\,(d+e\,x)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+i\sqrt{c}\right.$$

$$\left(\sqrt{c}\,d^2+i\sqrt{a}\,e^2\right)\sqrt{c\,d^4+a\,e^4}\,(d+e\,x)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(e^3\sqrt{c\,d^4+a\,e^4}\,\left(a+c\,x^4\right)+2\,(-1)^{1/4}a^{1/4}c^{3/4}d^2\sqrt{c\,d^4+a\,e^4}\,(d+e\,x)\sqrt{1+\frac{c\,x^4}{a}}\right.$$

$$\text{EllipticPi}\left[\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\text{ArcSin}\left[\frac{(-1)^{3/4}c^{1/4}x}{a^{1/4}}\right],-1\right]-c\,d^3\,e\,(d+e\,x)\sqrt{a+c\,x^4}$$

$$\left.\left.\text{Log}\left[-d^2+e^2x^2\right]+c\,d^3\,e\,(d+e\,x)\sqrt{a+c\,x^4}\,\text{Log}\left[a\,e^2+c\,d^2x^2+\sqrt{c\,d^4+a\,e^4}\,\sqrt{a+c\,x^4}\right]\right)\right)$$

## Problem 3: Unable to integrate problem.

$$\int\frac{1}{(d+e\,x)\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 518 leaves, 7 steps):

$$\frac{e\,\text{ArcTan}\left[\frac{\sqrt{-c\,d^4-b\,d^2e^2-a\,e^4}\,x}{d\,e\sqrt{a+b\,x^2+c\,x^4}}\right]}{2\sqrt{-c\,d^4-b\,d^2e^2-a\,e^4}}-\frac{e\,\text{ArcTanh}\left[\frac{b\,d^2+2\,a\,e^2+\left(2\,c\,d^2+b\,e^2\right)x^2}{2\sqrt{c\,d^4+b\,d^2e^2+a\,e^4}\,\sqrt{a+b\,x^2+c\,x^4}}\right]}{2\sqrt{c\,d^4+b\,d^2e^2+a\,e^4}}+$$

$$\left(c^{1/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(2\,a^{1/4}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\sqrt{a}\,\sqrt{c}\,d^2e^2},\right.\right.$$

$$\left.\left.2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(4\,a^{1/4}c^{1/4}d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 8, 26 leaves):

$$\int \frac{1}{(d+e\,x)\,\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

## Problem 4: Unable to integrate problem.

$$\int \frac{1}{(d+e\,x)^2\,\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 822 leaves, 11 steps):

$$-\frac{e^3\,\sqrt{a+b\,x^2+c\,x^4}}{\left(c\,d^4+b\,d^2\,e^2+a\,e^4\right)\,(d+e\,x)}+\frac{\sqrt{c}\,e^2\,x\,\sqrt{a+b\,x^2+c\,x^4}}{\left(c\,d^4+b\,d^2\,e^2+a\,e^4\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{d\,e\,\left(2\,c\,d^2+b\,e^2\right)\,\mathrm{ArcTan}\left[\frac{\sqrt{-c\,d^4-b\,d^2\,e^2-a\,e^4}\,x}{d\,e\,\sqrt{a+b\,x^2+c\,x^4}}\right]}{2\,\left(-c\,d^4-b\,d^2\,e^2-a\,e^4\right)^{3/2}}-$$

$$\frac{d\,e\,\left(2\,c\,d^2+b\,e^2\right)\,\mathrm{ArcTanh}\left[\frac{b\,d^2+2\,a\,e^2+\left(2\,c\,d^2+b\,e^2\right)\,x^2}{2\,\sqrt{c\,d^4+b\,d^2\,e^2+a\,e^4}\,\sqrt{a+b\,x^2+c\,x^4}}\right]}{2\,\left(c\,d^4+b\,d^2\,e^2+a\,e^4\right)^{3/2}}-$$

$$\left(a^{1/4}\,c^{1/4}\,e^2\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(\left(c\,d^4+b\,d^2\,e^2+a\,e^4\right)\,\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(c^{1/4}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(2\,a^{1/4}\,\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\,\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\,\left(2\,c\,d^2+b\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\mathrm{EllipticPi}\left[\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d^2\,e^2},2\,\mathrm{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

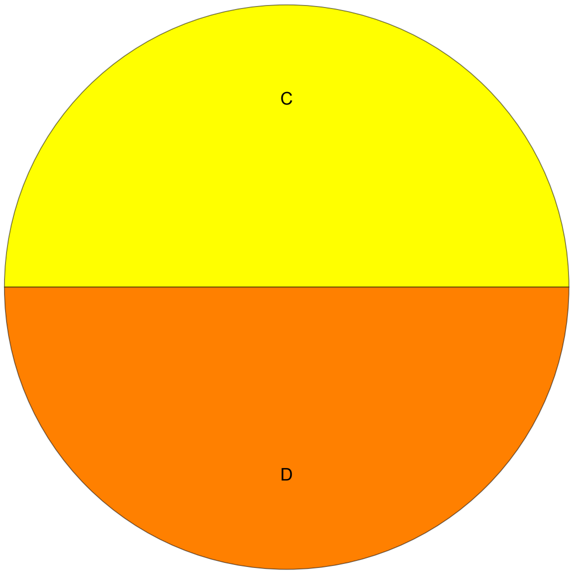
$$\left(4\,a^{1/4}\,c^{1/4}\,\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\,\left(c\,d^4+b\,d^2\,e^2+a\,e^4\right)\,\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 8, 26 leaves):

$$\int \frac{1}{(d+e\,x)^2\,\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

# Summary of Integration Test Results

4 integration problems




A - 0 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 2 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts